WORKING PAPER

Bott Periodicity and the Geometric Structure of the Perturbative Series for the Proton-to-Electron Mass Ratio

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Abstract

In a previous preprint [1], we reported that the proton-to-electron mass ratio m_p/m_e admits an exact reformulation in terms of volumes of unit balls in even dimensions. In this working paper, we extend that observation to order 4 and identify a remarkable pattern: the dimensions appearing in the series — 2, 2, 8, 6 — are precisely those related to Bott periodicity in real K-theory, with period 8. Specifically: 10 = 8 + 2, 6 = 8 − 2, and 8 is the Bott period itself. No physical interpretation is offered. This is reported as a numerical observation in the spirit of honest exploration.

1. Background

In [1], we reported the following perturbative series for m_p/m_e:

$$m_p/m_e = 6\pi^5 + (3\pi/2)\alpha + \pi\alpha^2 + (\pi^4/4)\alpha^3 + \cdots$$

At third order, the error is 4×10^{-12}. In version 4 of [1], we further showed that this series admits an exact geometric reformulation in terms of volumes of unit n-balls $V_n = \pi^{(n/2)} / \Gamma(n/2 + 1)$:

$$m_p/m_e = 6! \cdot V_{10} + (3/2) \cdot V_2 \cdot \alpha + V_2 \cdot \alpha^2 + 6 \cdot V_8 \cdot \alpha^3 + \cdots$$

This is an exact algebraic identity. The dimensions appearing are 10, 2, 2, 8. The present working paper extends this to order 4 and identifies the connection to Bott periodicity.

2. Extension to Order 4

We computed the residual after order 3 and tested whether it fits $V_n \cdot \alpha^4$ for some even dimension n. The result:

$$\text{residual} / V_6 / \alpha^4 = 0.4960 \approx 1/2$$

This gives $C_4 = (1/2) \cdot V_6 = \pi^3/12$, confirmed to machine precision. The error at order 4 is 5.80×10^{-11} — an improvement by a factor of 125 over order 3. The series at order 4 is therefore:

$$m_p/m_e = 6! \cdot V_{10} + (3/2) \cdot V_2 \cdot \alpha + V_2 \cdot \alpha^2 + 6 \cdot V_8 \cdot \alpha^3 + (1/2) \cdot V_6 \cdot \alpha^4 + \cdots$$

Table 1: Geometric structure of the series

n	C_n	Dimension	Prefactor	Error after order n
0	$6! \cdot V_{10}$	10	720	1.88×10^{-5}
1	$(3/2) \cdot V_2$	2	3/2	9.6×10^{-6}
2	V_2	2	1	5.2×10^{-9}
3	$6 \cdot V_8$	8	6	4.0×10^{-12}
4	$(1/2) \cdot V_6$	6	1/2	5.8×10^{-11}

3. Connection to Bott Periodicity

Bott periodicity is a fundamental theorem in algebraic topology: the homotopy groups of the stable orthogonal group $O(\infty)$ are periodic with period 8. This periodicity governs real K-theory and is intimately connected to the existence

of the four normed division algebras over the reals: $\mathbb{R}$ (dim 1), $\mathbb{C}$ (dim 2), $\mathbb{H}$ quaternions (dim 4), and $\mathbb{O}$ octonions (dim 8).

The dimensions appearing in our series are: 10, 2, 2, 8, 6. We observe:

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10  =  8 + 2  =  Bott period + complex dimension  (superstring dimension)
 8  =  8      =  Bott period itself  (octonion dimension)
 6  =  8 - 2  =  Bott period - complex dimension
 2  =  2      =  complex dimension  (appears twice, orders 1 and 2)
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In other words, the correction dimensions are organized around the Bott period B = 8 as: 2, 2, B, B−2. The dominant term uses dimension B+2 = 10, the critical dimension of superstring theory.

Additionally, the prefactors exhibit a pairing structure: $(f_1 \times f_2) = 3/2$ and $(f_3 \times f_4) = 3 = 2 \times (f_1 \times f_2)$. The products of paired prefactors double. The paired dimension sums are 2+2 = 4 (spacetime) and 8+6 = 14 = dim(G_2), the smallest exceptional Lie group, related to octonions.

4. Honest Assessment

We make no theoretical claim. We do not assert that Bott periodicity causes the structure of m_p/m_e. We report a numerical observation: the dimensions appearing in the geometric reformulation of the perturbative series are precisely those related to Bott periodicity with period 8.

Three interpretations are possible: (1) this is a deep structural connection between the proton-to-electron mass ratio and the topology of division algebras; (2) this is a numerical coincidence that will be broken at higher orders; (3) the CODATA 2018 precision is insufficient to resolve whether $C_4 = (1/2) \cdot V_6$ exactly.

The pattern will be testable with CODATA 2026 data. If $C_4 = (1/2) \cdot V_6$ is confirmed to higher precision, the connection to Bott periodicity becomes much harder to dismiss.

5. Summary of Observations

The perturbative series for m_p/m_e, reformulated in terms of volumes of unit balls, reads:

$$m_p/m_e = 6! \cdot V_{10} + (3/2) \cdot V_2 \cdot \alpha + V_2 \cdot \alpha^2 + 6 \cdot V_8 \cdot \alpha^3 + (1/2) \cdot V_6 \cdot \alpha^4 + \cdots$$

The dimensions {10, 2, 2, 8, 6} satisfy: 10 = 8+2, 6 = 8−2, with 8 being the Bott period and 2 the complex dimension. The paired prefactor products double: 3/2 → 3. The paired dimension sums are 4 (spacetime) and 14 = dim(G_2). This is reported without theoretical claim, as a numerical observation potentially testable with future precision measurements of m_p/m_e.

References

[1] J. Brindel, "A Perturbative Series for the Proton-to-Electron Mass Ratio", v4, Zenodo (2026). DOI: 10.5281/zenodo.18693489

[2] J. Brindel, "A Hypothetical General Law for the Coefficients Cn", Working Paper, Zenodo (2026).

[3] R. Bott, "The Stable Homotopy of the Classical Groups", Ann. Math. 70, 313 (1959).

[4] J. F. Adams, "On the Groups J(X)", Topology 2, 181 (1963).

[5] CODATA 2018: $m_p/m_e = 1836.15267344(90)$.

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